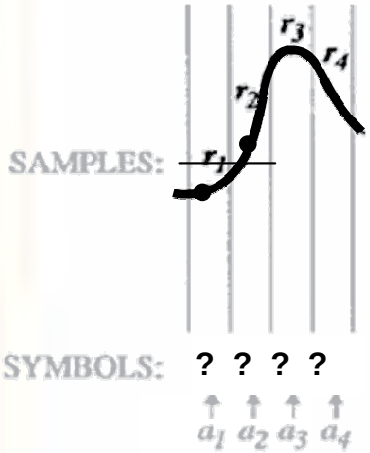
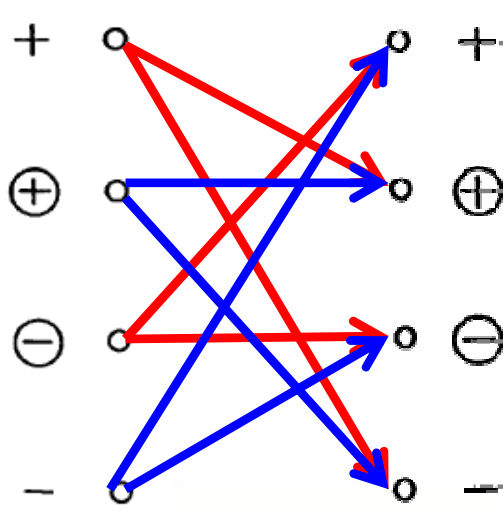
Maximum Likelihood Sequence Detection

- Step-by-step process to find a sequence of symbols that matches the signal samples



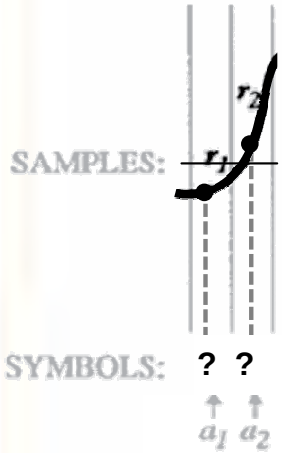
- The trellis is a guide to possible paths
- The goal is the maximum likelihood sequence

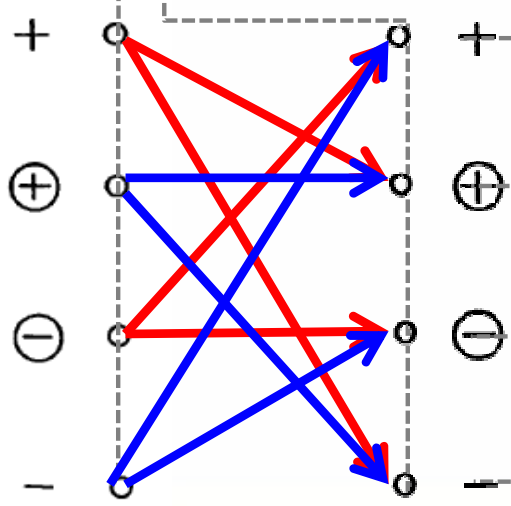


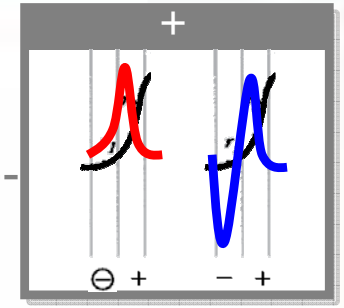
+	⊕	⊖	–
must follow ⊖ or –	must follow ⊕ or +	must follow ⊖ or –	must follow ⊕ or +

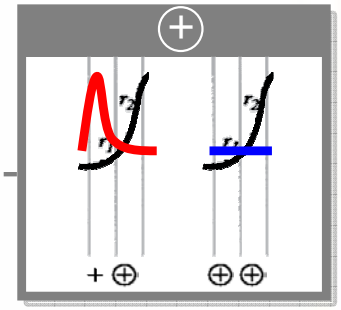
Maximum Likelihood Sequence Detection

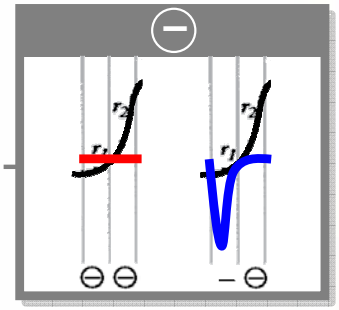
- Step-by-step process to find a sequence of symbols that matches the signal samples
  - Each path corresponds to an ideal signal
  - Start with the first two samples

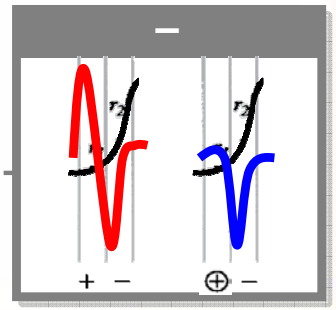






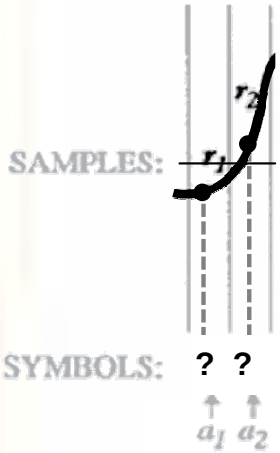




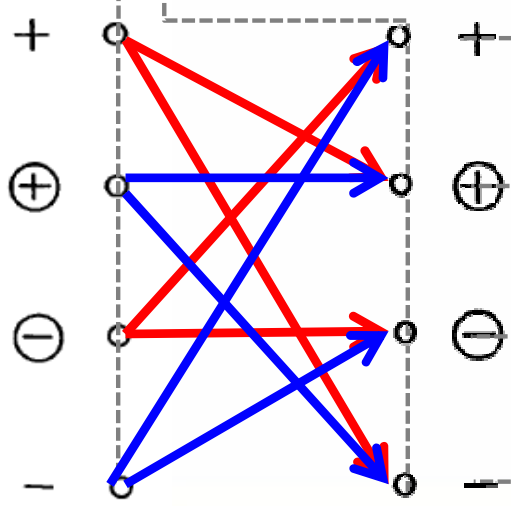


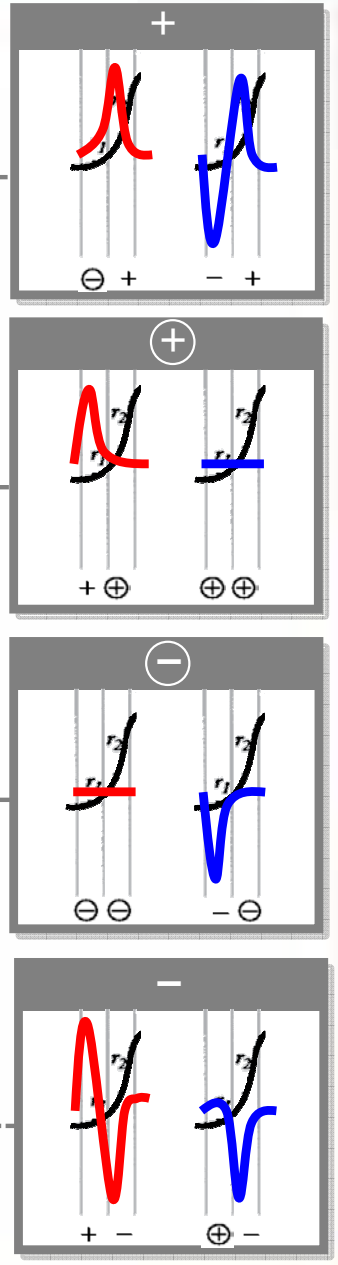
Maximum Likelihood Sequence Detection

- Eliminate half the paths by comparing pairs



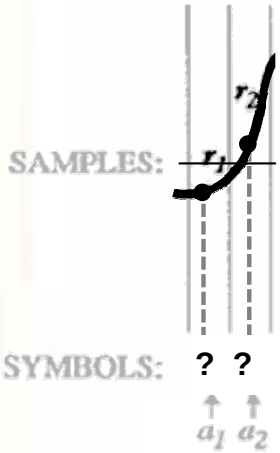
- Compare paths leading to the same symbol
- Retain the more likely path in each pair



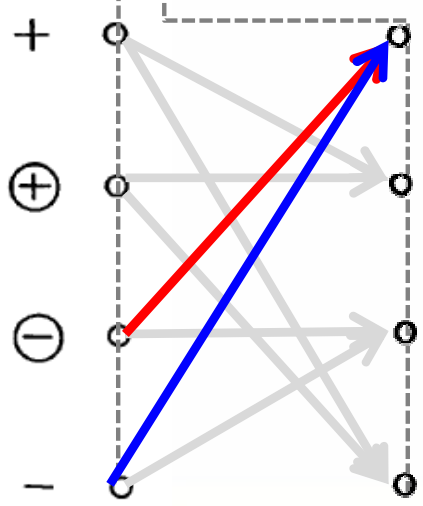


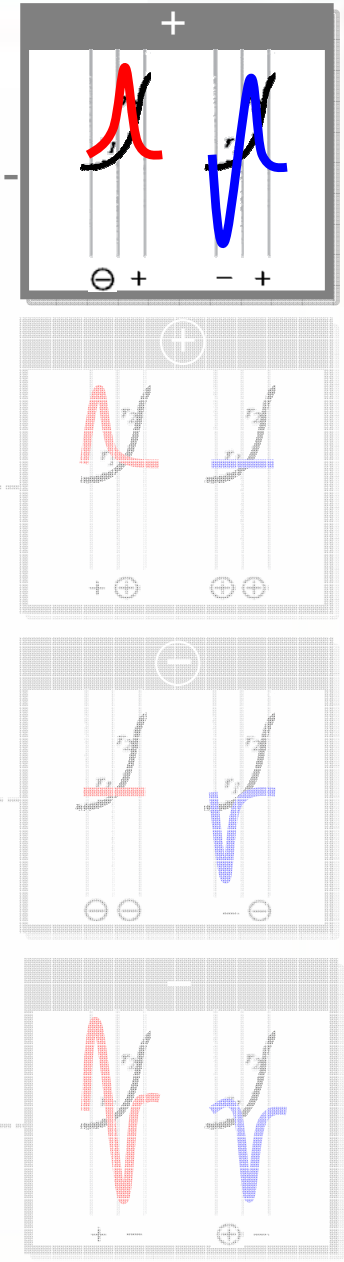
Maximum Likelihood Sequence Detection

- Compare two paths ending in +



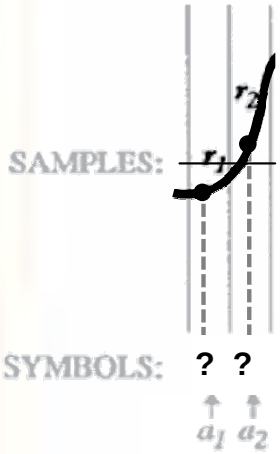
- Which signal is a better match for the samples?
- Retain the more likely path



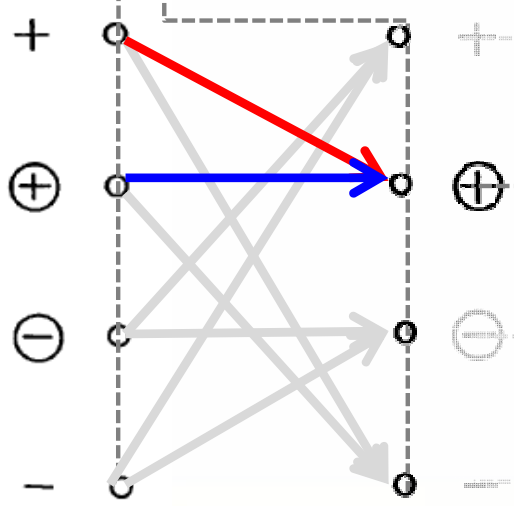


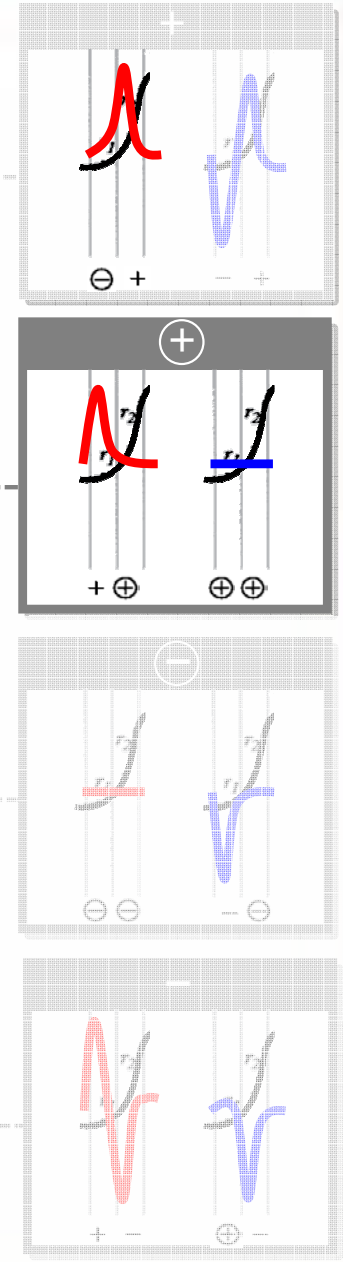
Maximum Likelihood Sequence Detection

- Compare two paths ending in ⊕



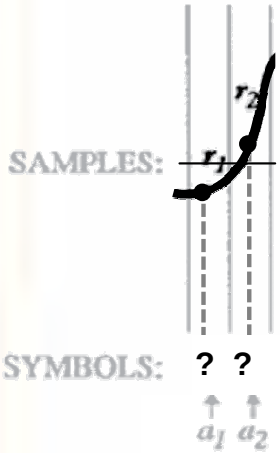
- Which signal is a better match for the samples?
- Retain the more likely path



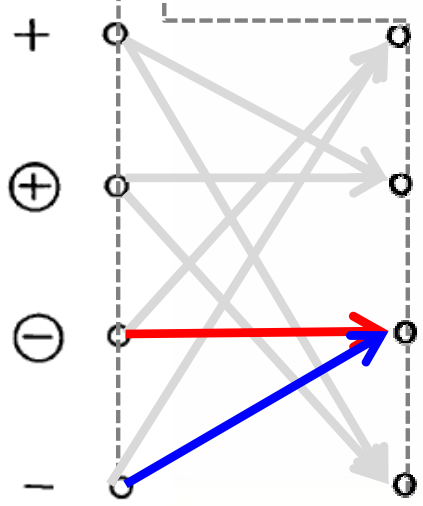


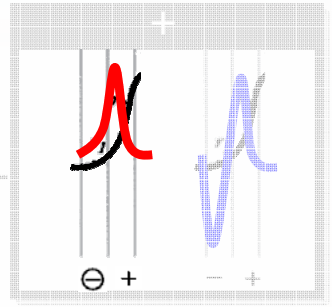
Maximum Likelihood Sequence Detection

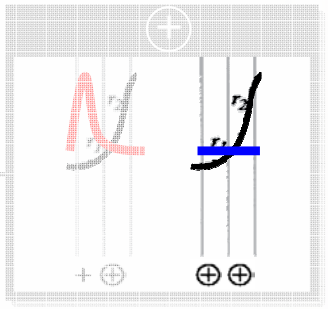
- Compare two paths ending in ⊖

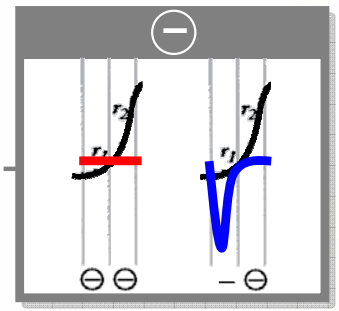


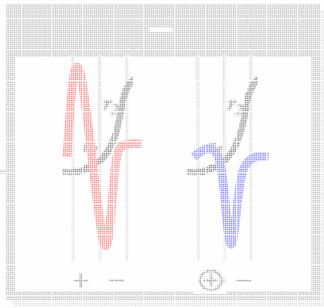
- Which signal is a better match for the samples?
- Retain the more likely path





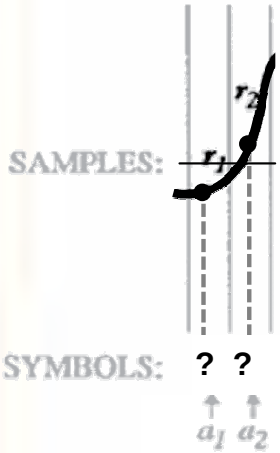




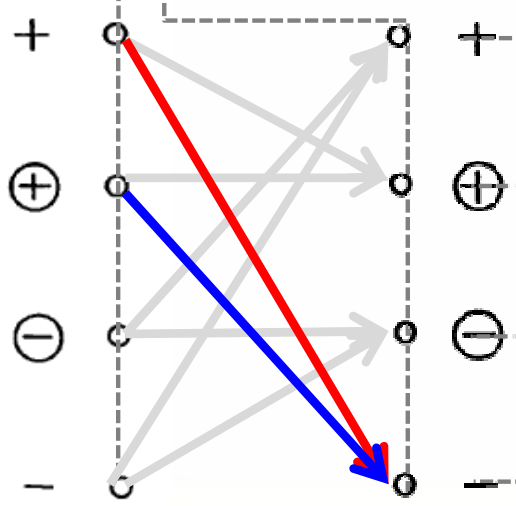


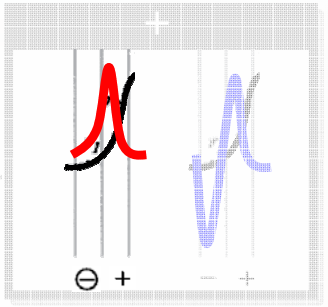
Maximum Likelihood Sequence Detection

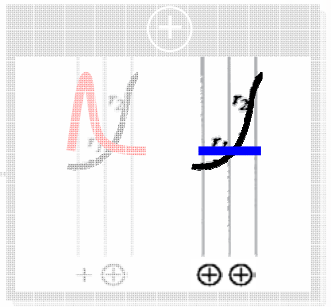
- Compare two paths ending in –

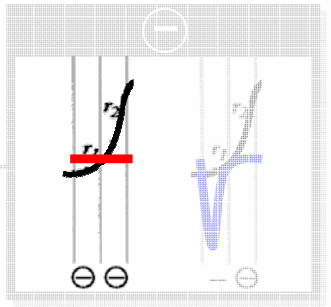


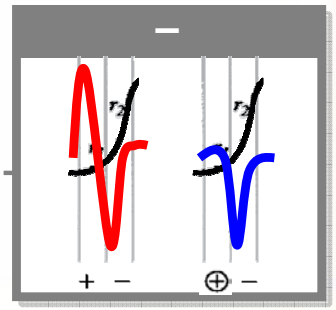
- Which signal is a better match for the samples?
- Retain the more likely path





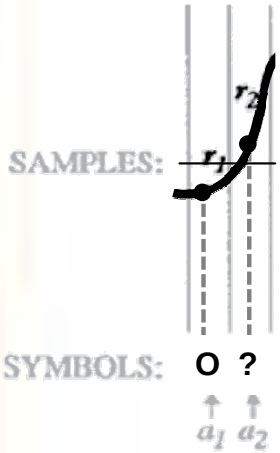




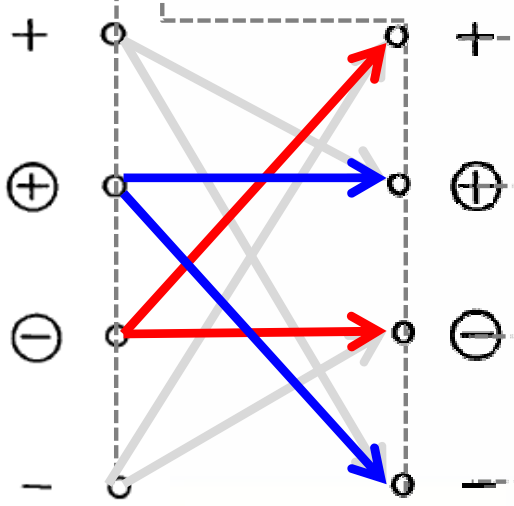


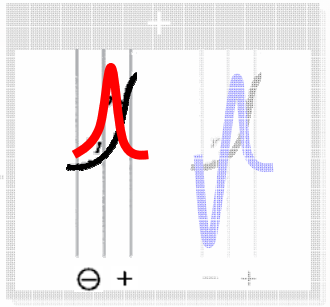
Maximum Likelihood Sequence Detection

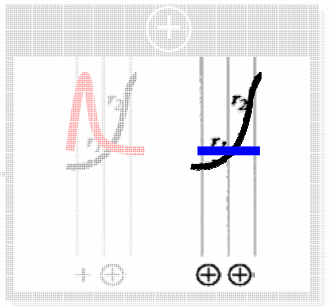
- Retain only four possible paths

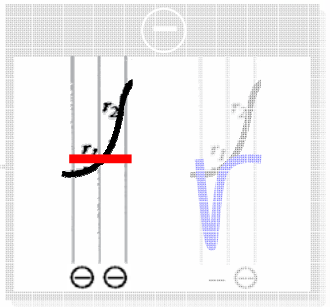


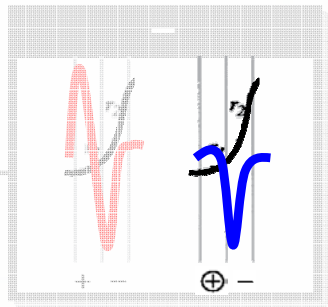
- + and – have been eliminated for the first symbol
- First symbol must be ⊕ or ⊖





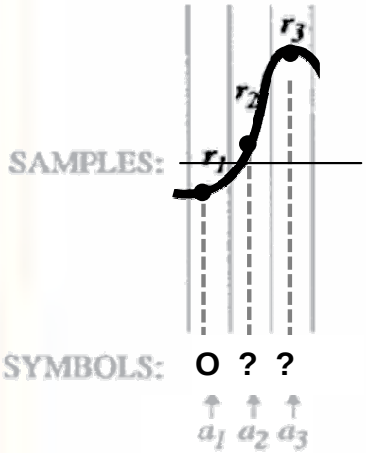




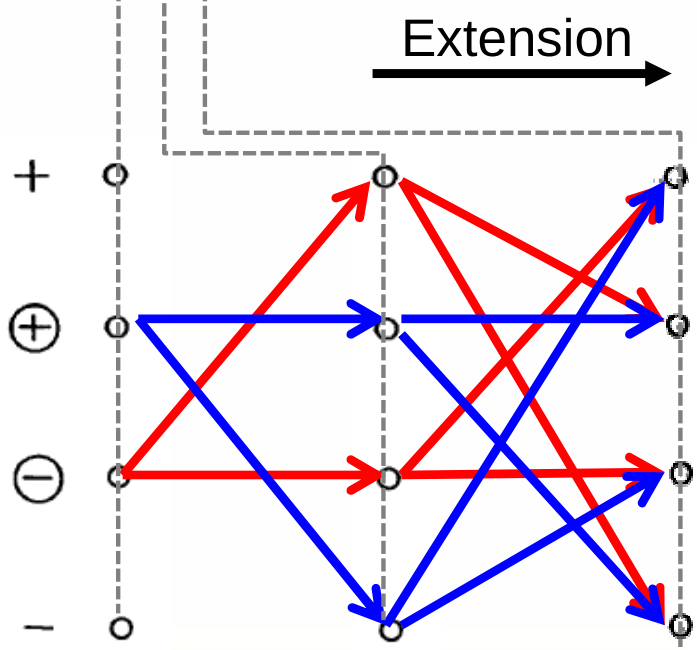


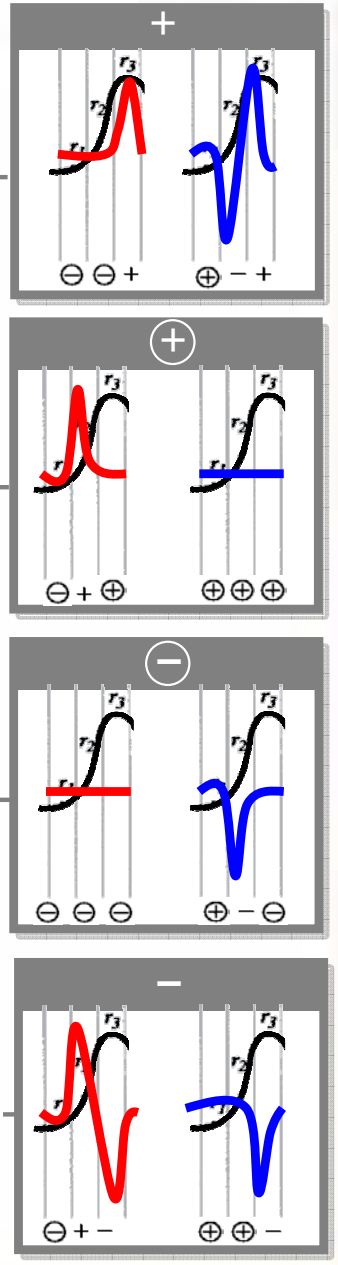
Maximum Likelihood Sequence Detection

- Extend surviving paths to next sample



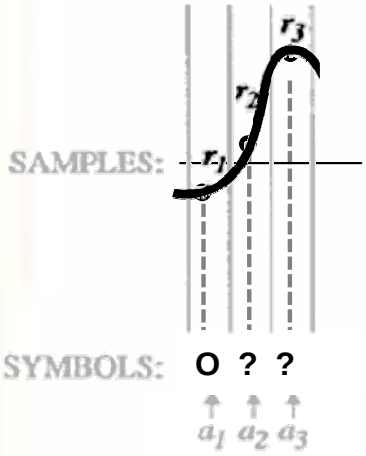
- Eight new branches define new paths
- Repeat comparisons for each node



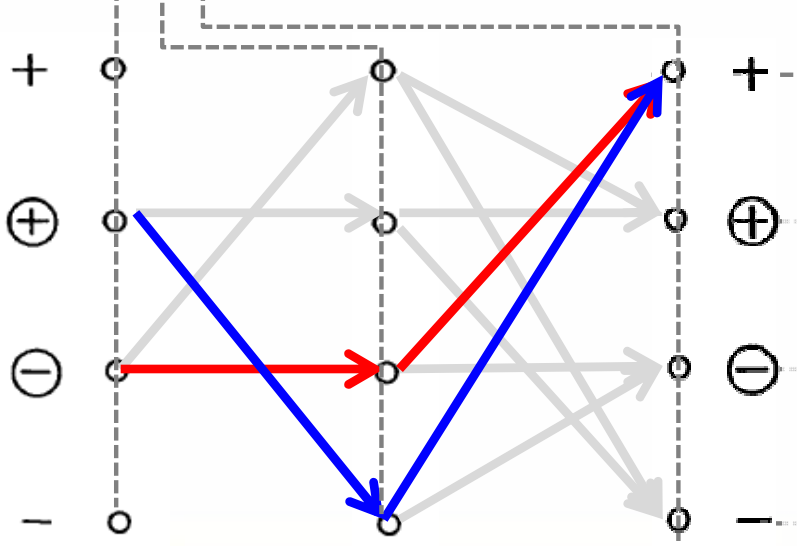


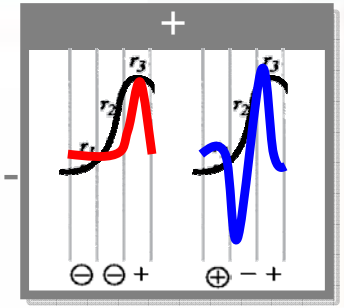
Maximum Likelihood Sequence Detection

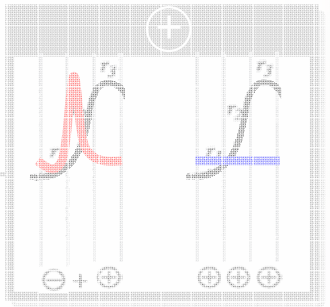
- Compare two paths ending in +

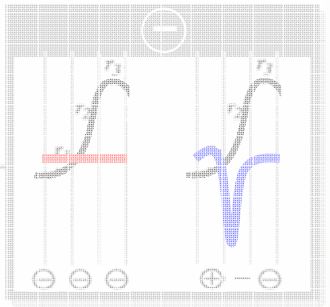


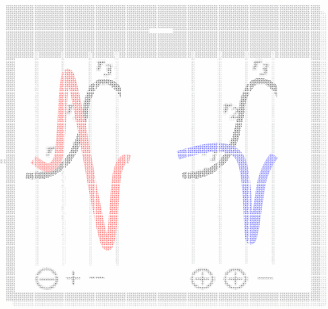
- Which signal is a better match for the samples?
- Retain the more likely path





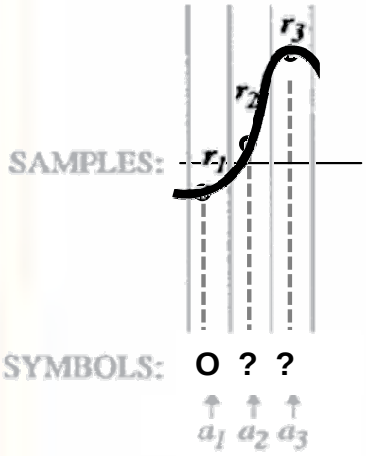




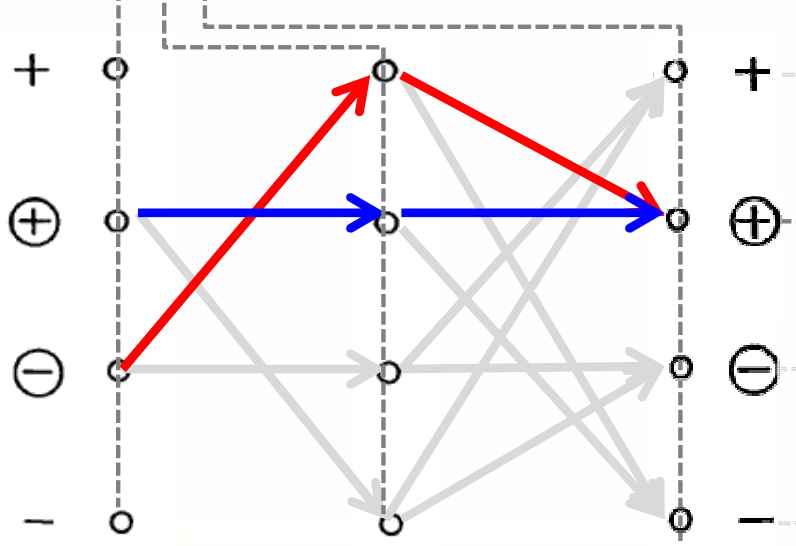


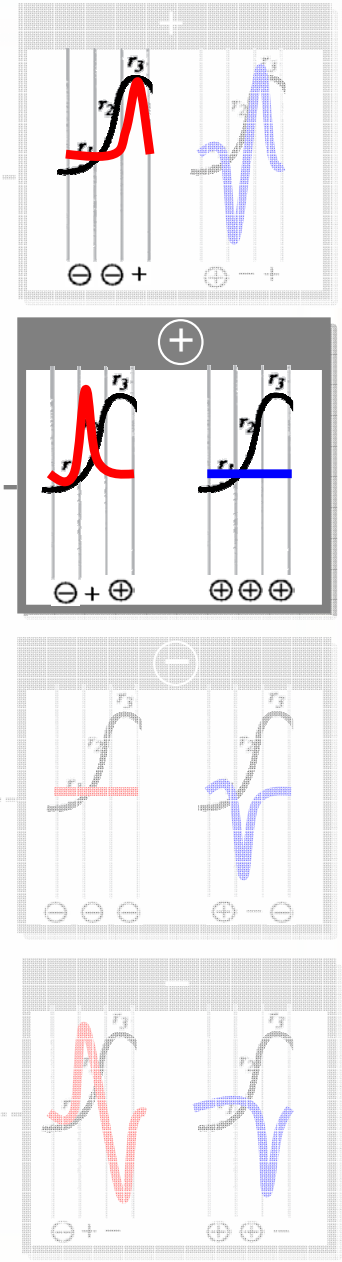
Maximum Likelihood Sequence Detection

- Compare two paths ending in ⊕



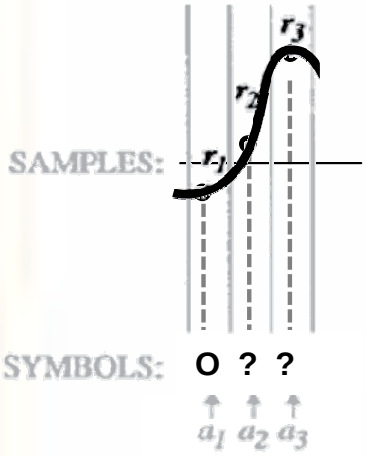
- Which signal is a better match for the samples?
- Retain the more likely path



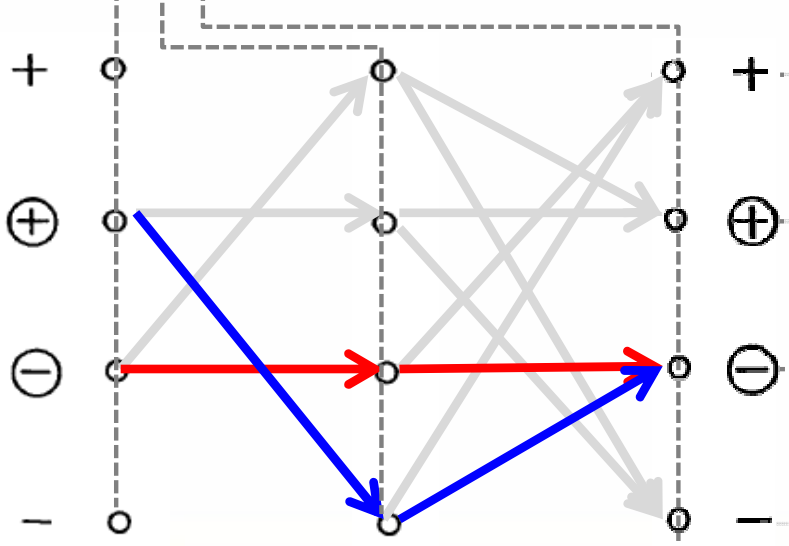


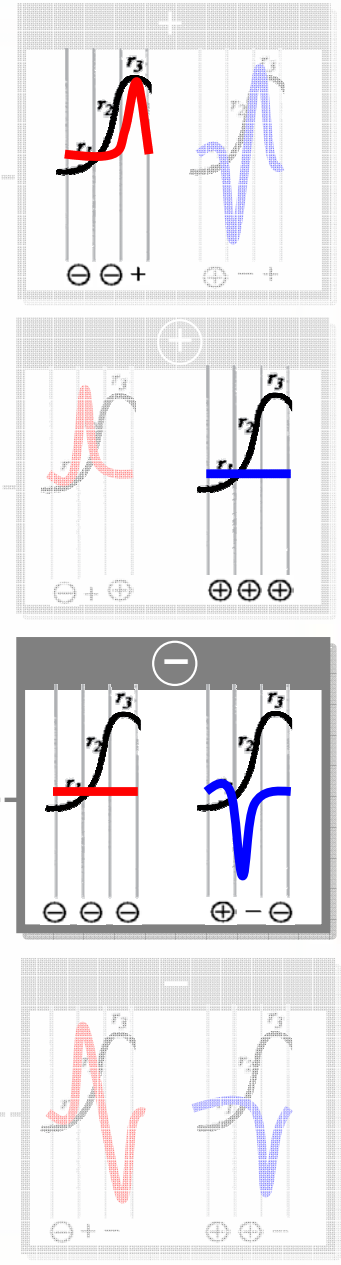
Maximum Likelihood Sequence Detection

- Compare two paths ending in ⊖



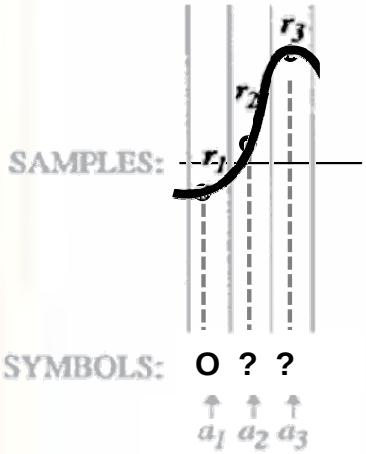
- Which signal is a better match for the samples?
- Retain the more likely path



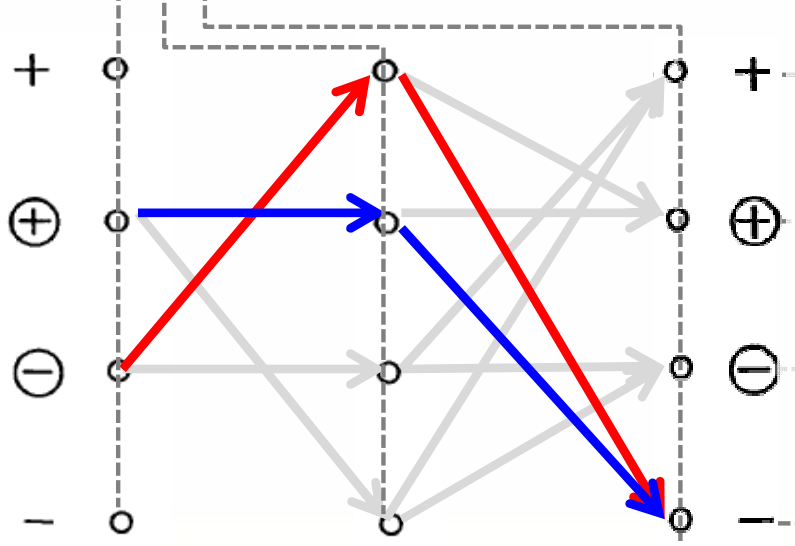


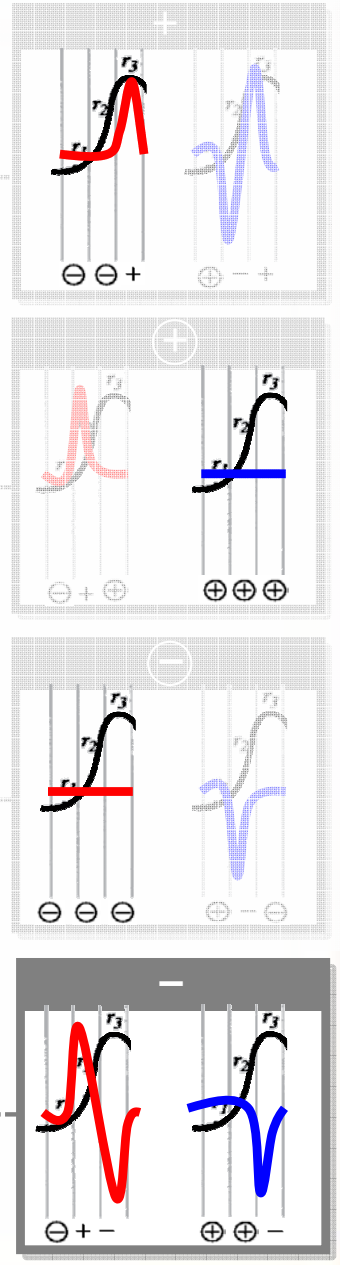
Maximum Likelihood Sequence Detection

- Compare two paths ending in –



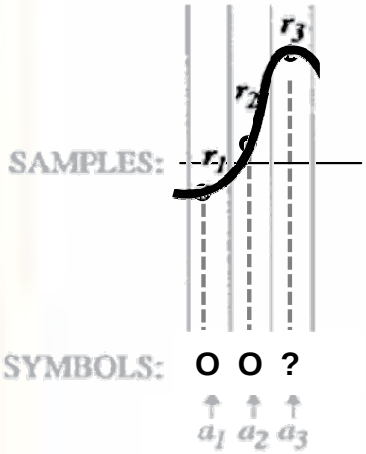
- Which signal is a better match for the samples?
- Retain the more likely path



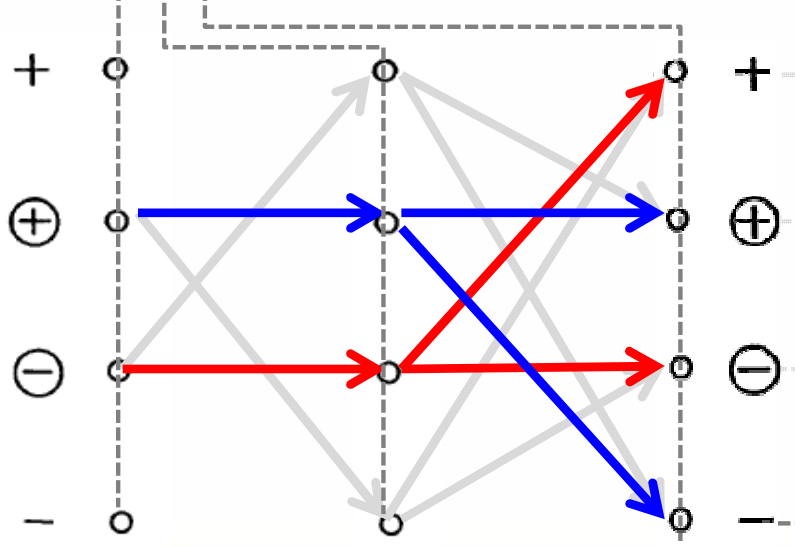


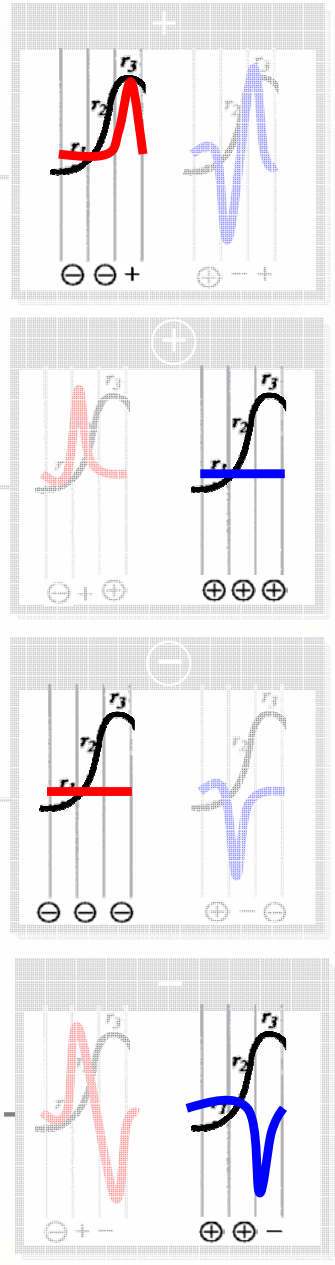
Maximum Likelihood Sequence Detection

- Retain only four possible paths



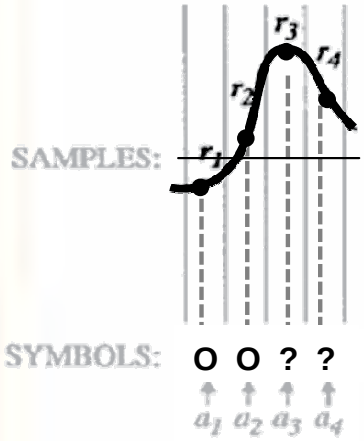
- + and – have been eliminated for both first and second symbol
- First two symbols must be ⊕⊕ or ⊖⊖



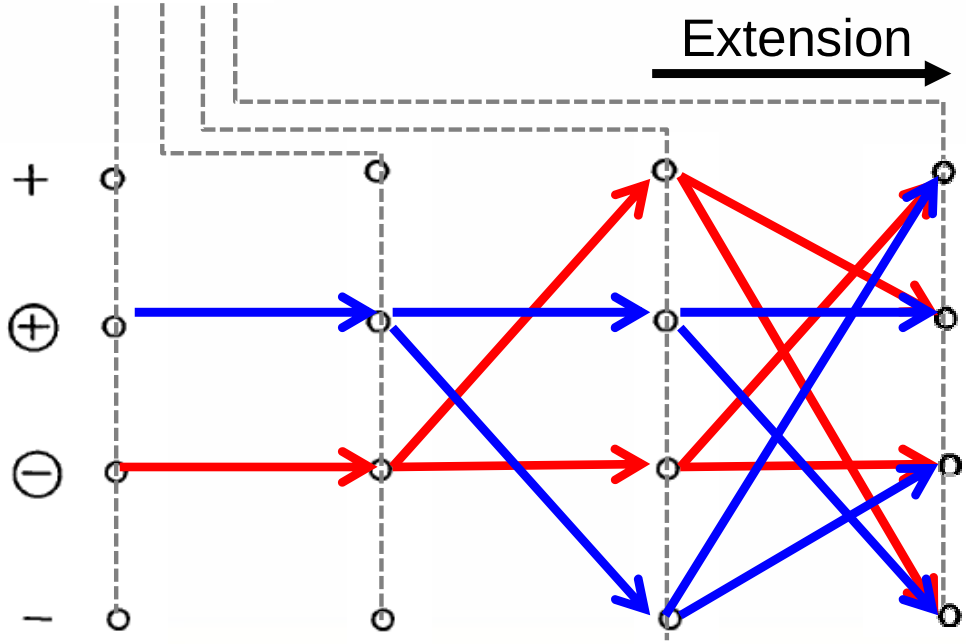


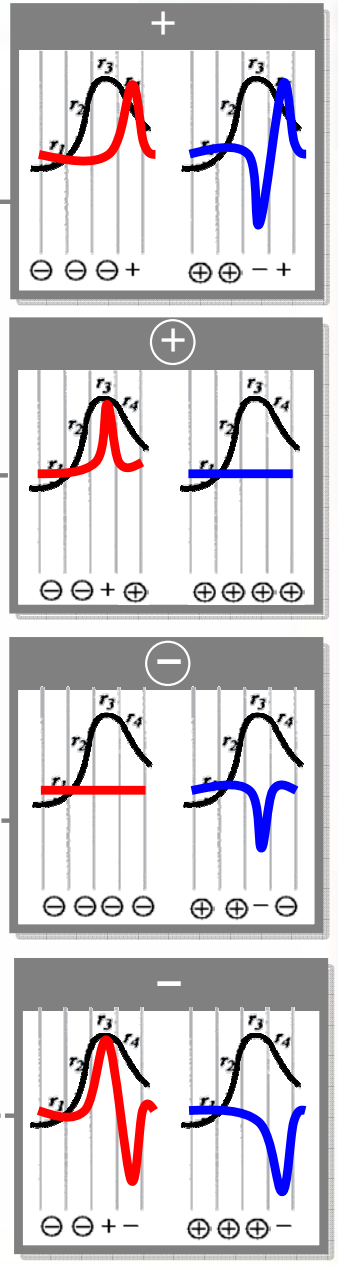
Maximum Likelihood Sequence Detection

- Extend surviving paths to next sample



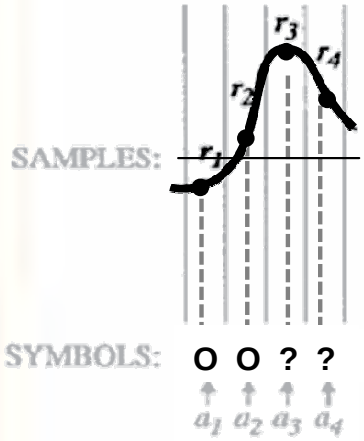
- Eight new branches define new paths
- Repeat comparisons for each node



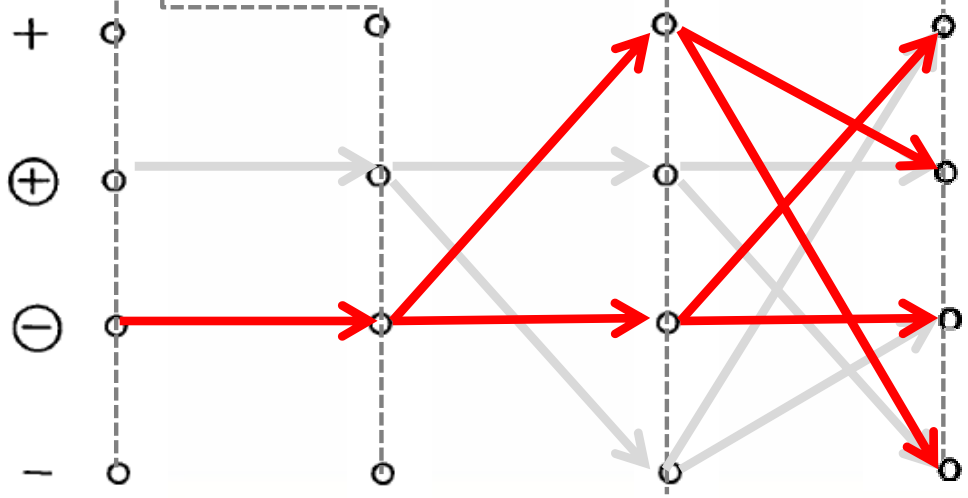


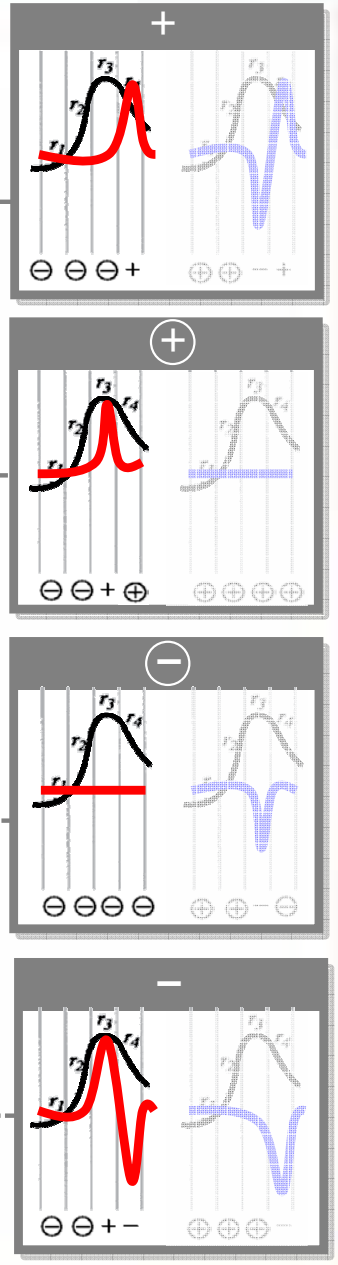
Maximum Likelihood Sequence Detection

- Eliminate half the paths by comparing pairs



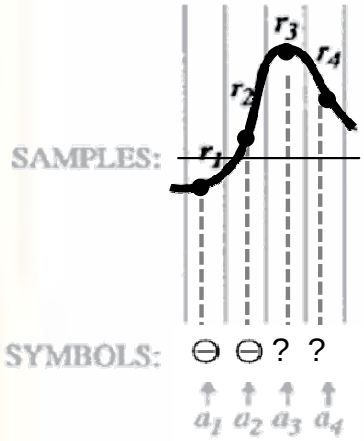
- Compare paths leading to the same symbol
- The red path is the most likely in each pair



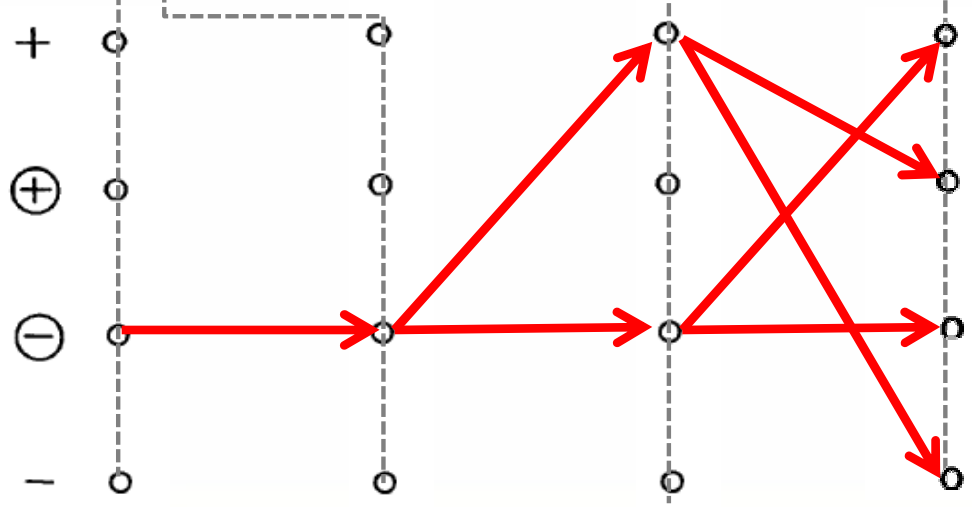


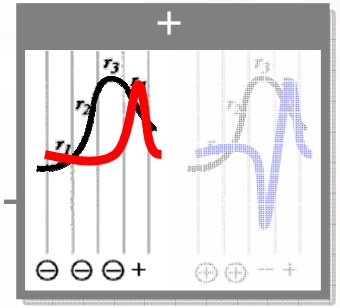
Maximum Likelihood Sequence Detection

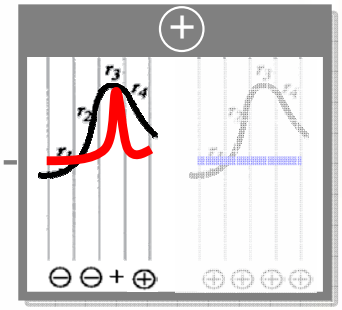
- Retain only four possible paths

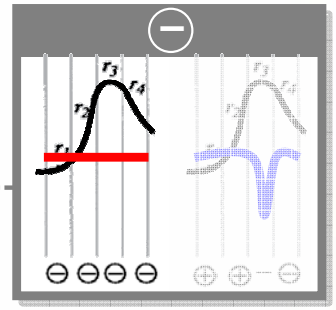


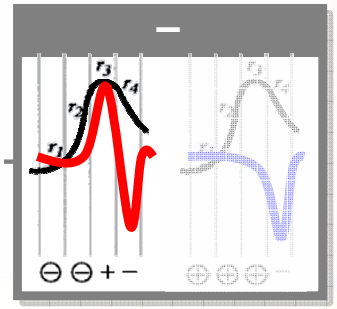
- First two symbols must be ⊖ ⊖
- Third symbol must be + or ⊖





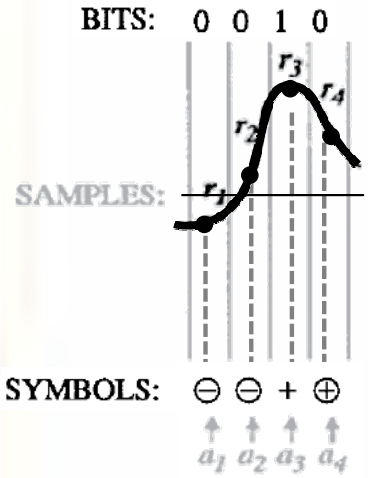




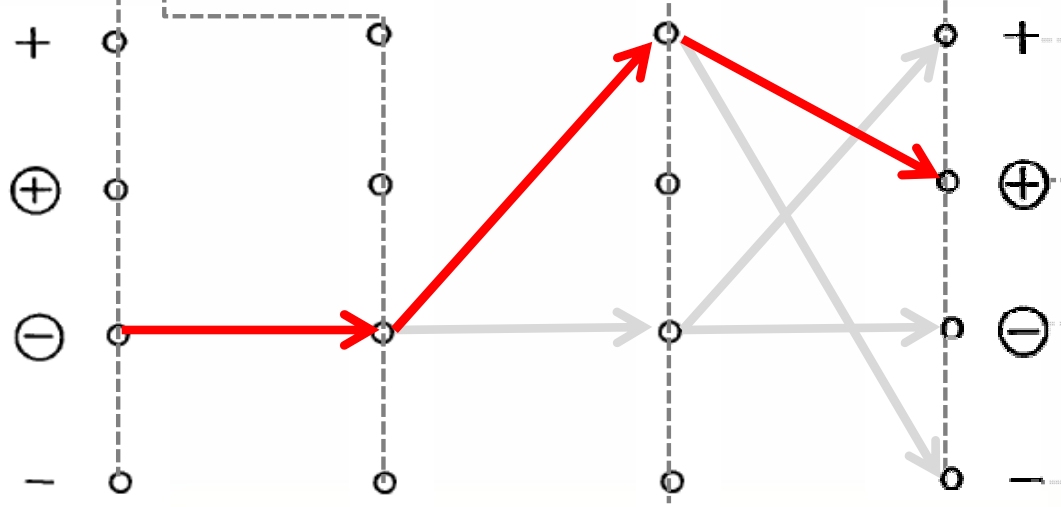


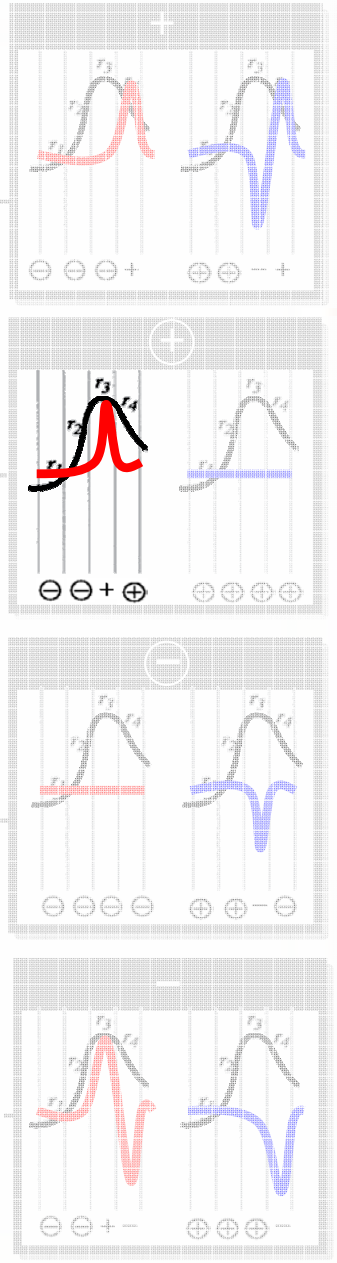
Maximum Likelihood Sequence Detection

- Four surviving paths may be compared



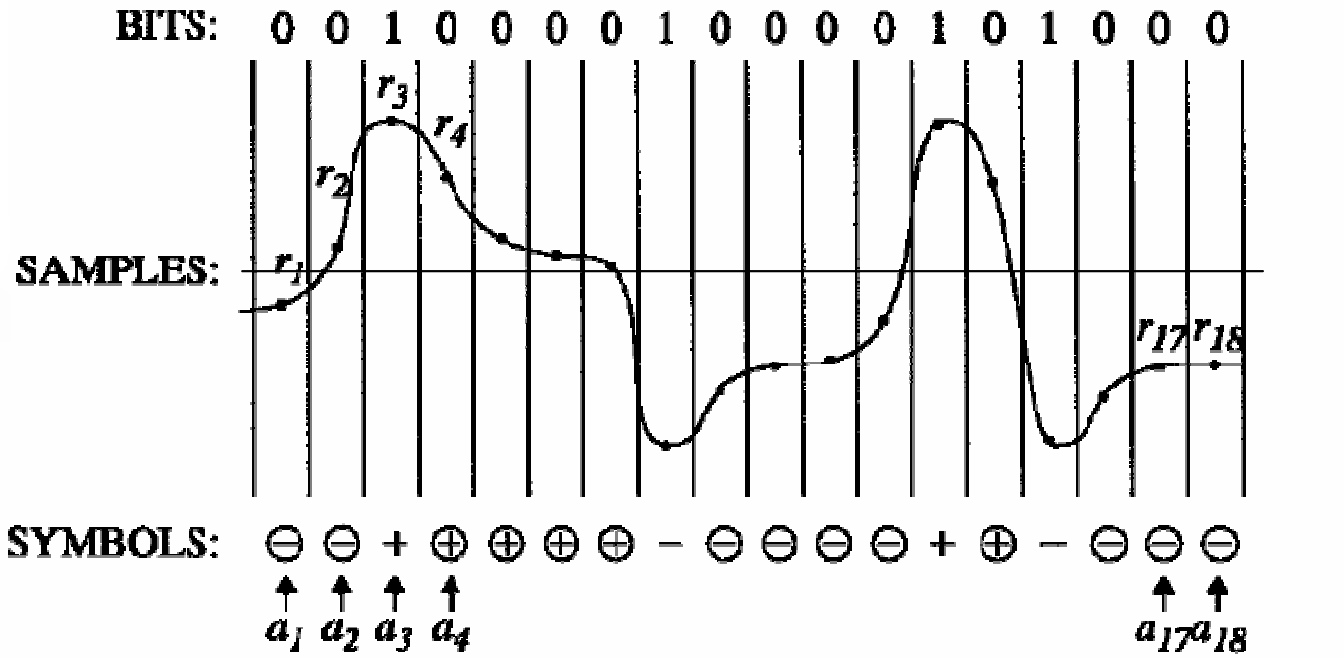
- If this is the end of the data, the most likely path of the four is the final sequence
- ⊖⊖+⊕ is the most likely sequence of symbols: digital output is 0 0 1 0

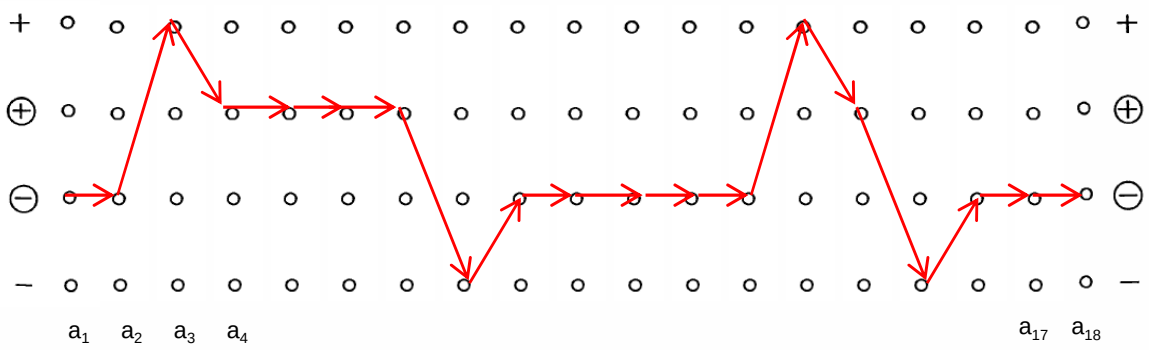




Maximum Likelihood Sequence Detection

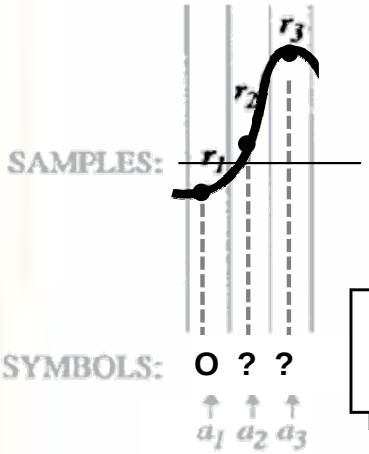
- Sequence detection can proceed for all samples





Sequence Detection – Branch Metrics

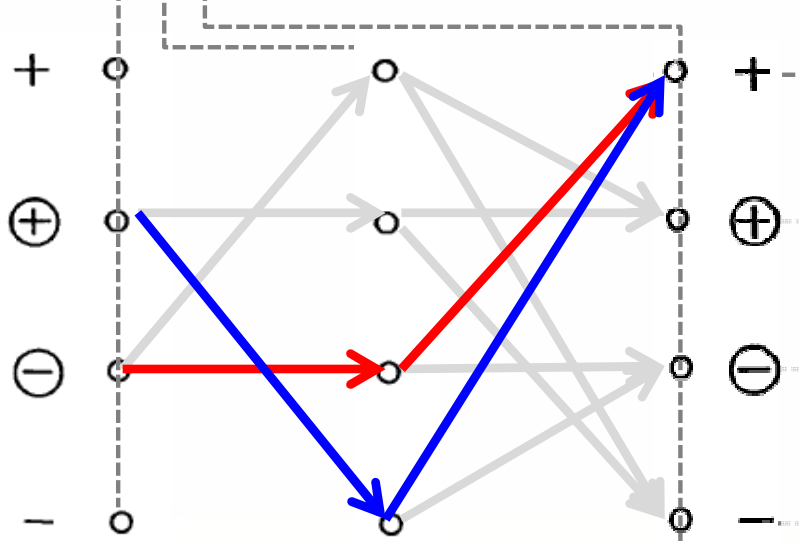
- Electronics use math to compare paths

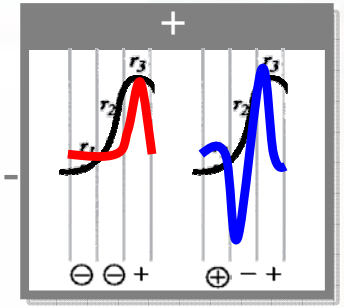


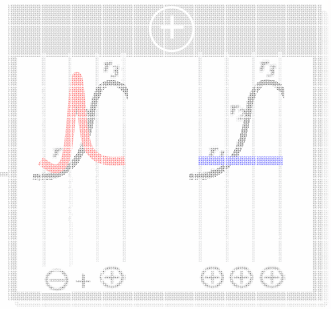
- A numerical value called a branch metric measures how well a branch matches each sample

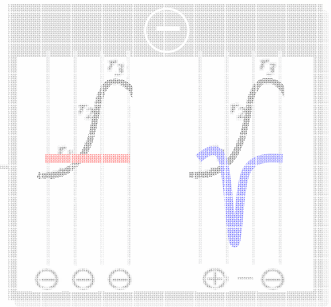
"Branch metric value," "branch metric," and "metric value" mean "the numerical value of a 'branch.'"

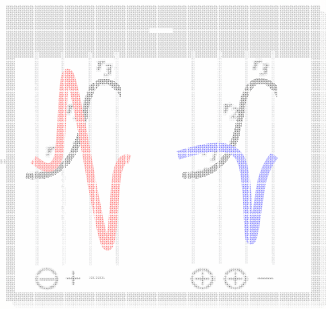
Joint Agreed Terms (Dkt. 74)











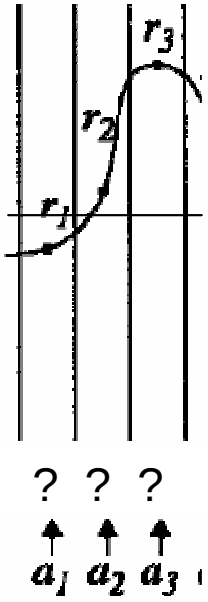
Accumulated Branch Metrics

- Simple Branch Metric: measure differences

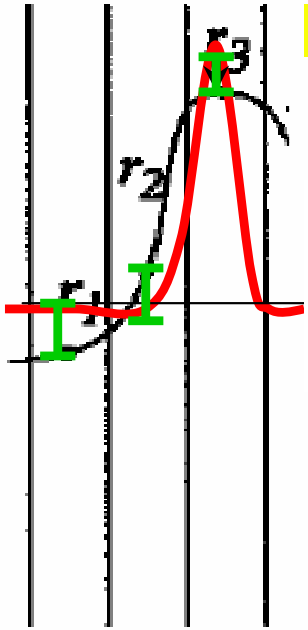
$r_3 = 0.9$

$r_2 = 0.2$

$r_1 = -0.2$



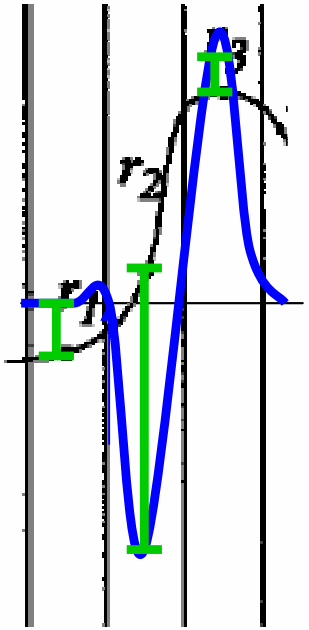
$M_i = | r_i - m_i |$



$m_3 = 1$

$r_3 = 0.9$

0.2+0.2+0.1 =0.5



$m_3 = 1$

$r_3 = 0.9$

0.2+1.2+0.1 =1.5

- Add (accumulate) the branch metric values to compare
- The smaller total is the best accumulated metric

A "branch metric function" is "a mathematical function for determining a 'branch metric value' for a 'branch.'"

Joint Agreed Terms (Dkt. 74)

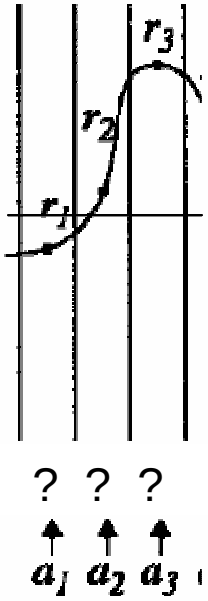
Accumulated Branch Metrics

- More complex branch metrics can be used

$r_3 = 0.9$

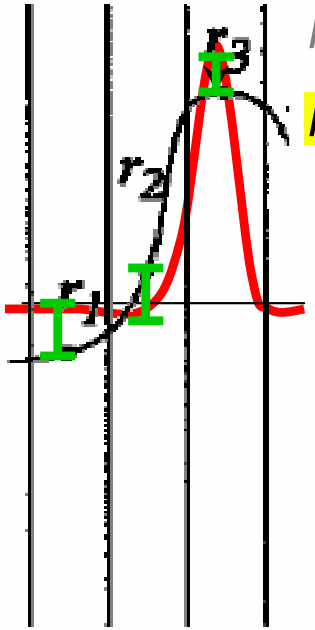
$r_2 = 0.2$

$r_1 = -0.2$

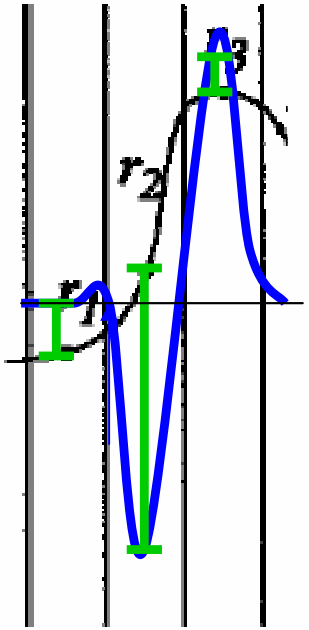


$M_i = | r_i - m_i |$

$M_i = (r_i - m_i)^2$



$(0.2)^2+(0.2)^2+(0.1)^2 = 0.09$



$(0.2)^2+(1.2)^2+(0.1)^2 = 1.49$

- Formulas for branch metrics can account for different types of noise

Sequence Detection – Branch Metrics

- Three Branch Metrics described in patents:
  - "Euclidean"

$$M_i = N_i^2 = (r_i - m_i)^2 \qquad (8)$$

  - "Variance Dependent"

$$M_i = \log\sigma_i^2 + \frac{N_i^2}{\sigma_i^2} = \log\sigma_i^2 + \frac{(r_i - m_i)^2}{\sigma_i^2} \qquad (10)$$

  - "Correlation-Sensitive"

$$M_i = \log \det\frac{C_i}{\det\ c_i} + \underline{N}_i^T C_i^{-1} \underline{N}_i - \underline{n}_i^T c_i^{-1} \underline{n}_i \qquad (13)$$

'839 Patent cols. 5-7